INVESTIGATION ON INFLUENCE

OF PROPOSED

INTERNATIONAL PASSAMAQUODDY TIDAL POWER PROJECT

ON TIDES IN THE BAY OF FUNDY

Prepared for

Passamaquoddy Tidal Power Survey
U.S. Army Engineer Division, New England
Corps of Engineers, 150 Causeway Street
Boston 14, Massachusetts

Under Contract No. DA-19-016-CIV-ENG-58-174

by

ARTHUR T. IPPEN and DONALD R. F. HARLEMAN

Consulting Engineers
M.I.T., Cambridge, Massachusetts

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FOREWORD

The study reported herein was conducted for the Passamaquoddy Tidal Power Survey, U. S. Army Engineer Division, New England, Corps of Engineers, under Contract No. DA-19-016-CIV-ENG-58-174, authorized under Passamaquoddy Tidal Power Project PL 401-84th Congress, 2nd Session, approved 31 January 1956 and 10 U.S.C. 2304 (a) (4).

The authors wish to express their appreciation to Mr. R. D. Field, Chief, Passamaquoddy Survey Division and to Mr. Lincoln Reid for furnishing the information and tidal data necessary for the investigation.

I. INTRODUCTION

The general nature of the study is concerned with an analytical investigation of the effect of the proposed Passamaquoddy Tidal Power project on the existing tides in the vicinity of Eastport, Maine. Inasmuch as the engineering, economic and feasibility survey is based upon the existing tidal regime, it is of considerable importance to determine whether the proposed tidal power structures will significantly alter the tides after their construction.

The general plan for the project structures is shown in Figure 1. An upper pool is formed by closing Passamaquoddy Bay from the Bay of Fundy by a series of dikes. The upper pool is filled during a portion of each high tide cycle by 40 gates located between the mainland of New Brunswick and Macmaster Island and by 50 gates between Deer Island and Eastport, Maine. A lower pool is formed by a series of dikes enclosing Cobscook Bay. The lower pool is emptied on a portion of the low tide cycle by a single set of 70 gates located between Campobello and Indian Islands. A thirty unit power house, located at Eastport, generates power on a continuous basis by discharging flow from the upper to the lower pool.

For the purposes of this investigation the southern limit of the Bay of Fundy, or junction with the Gulf of Maine, is defined by a line drawn between Cutler, Maine, and Grand Passage, St. Mary Bay, Nova Scotia. The Gulf of Maine may be roughly defined as joining the Atlantic Ocean along a line between Chatham on Cape Cod and Cape Sable, Nova Scotia. The longitudinal axis of the Bay of Fundy lies in a north-easterly direction and at a point, approximately 110 miles from Eastport, the Bay divides into two arms, Chignecto Bay and Minas Basin. A map of the entire Bay of Fundy showing the geographical features discussed and the location of the tidal stations used in the study is shown in Figure 2.

The general objectives of the study are summarized below:

Objectives of Study

1. To develop a simplified mathematical model of the Bay of Fundy in which the tidal motions are simulated in reasonable agreement

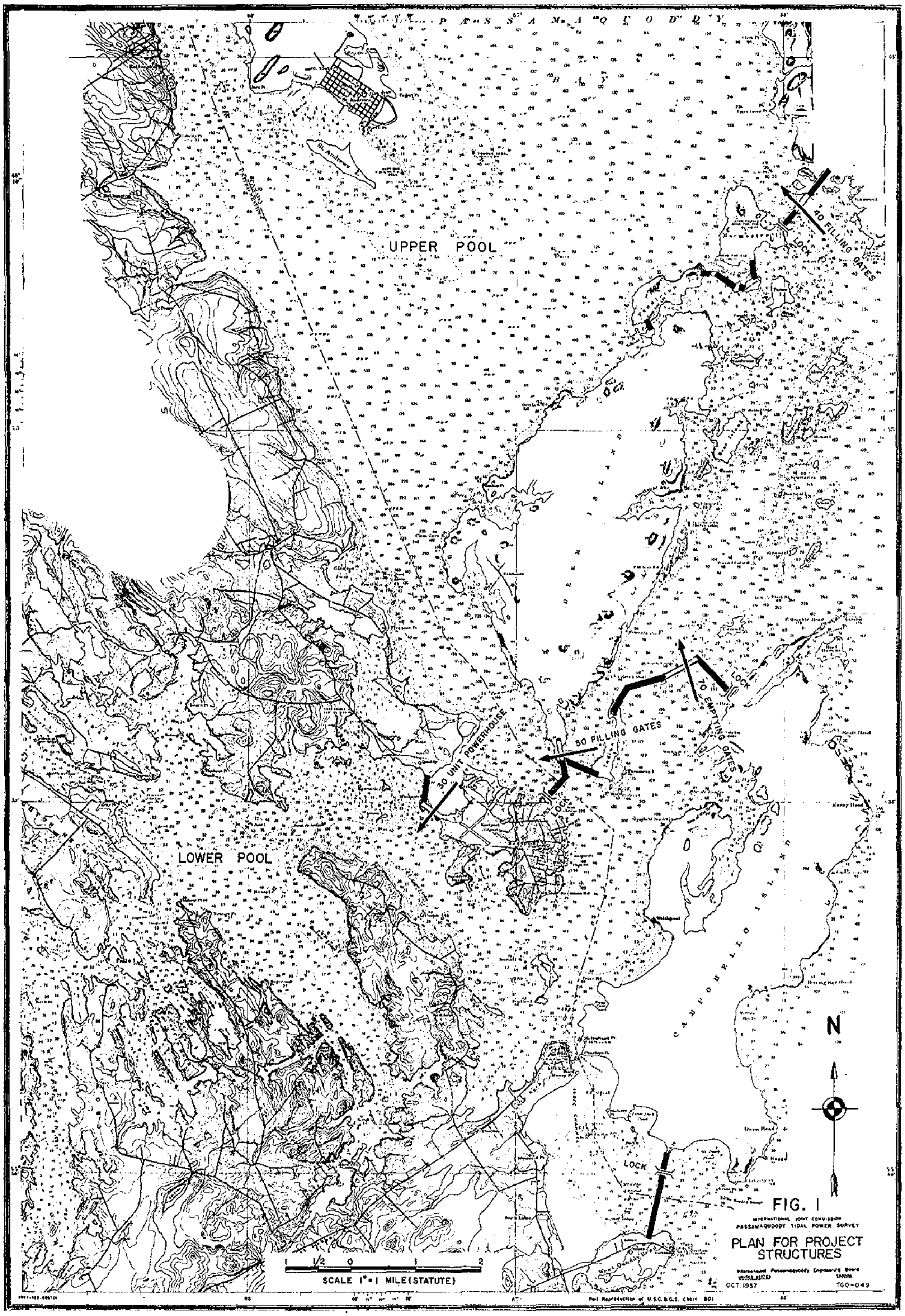

FIG. I

INTERNATIONAL JOINT COMMISSION
PASSAMAQUODDY TIDAL POWER SURVEY
PLAN FOR PROJECT STRUCTURES
International Passamaquoddy Engineering Board
OCT. 1957 T00-049

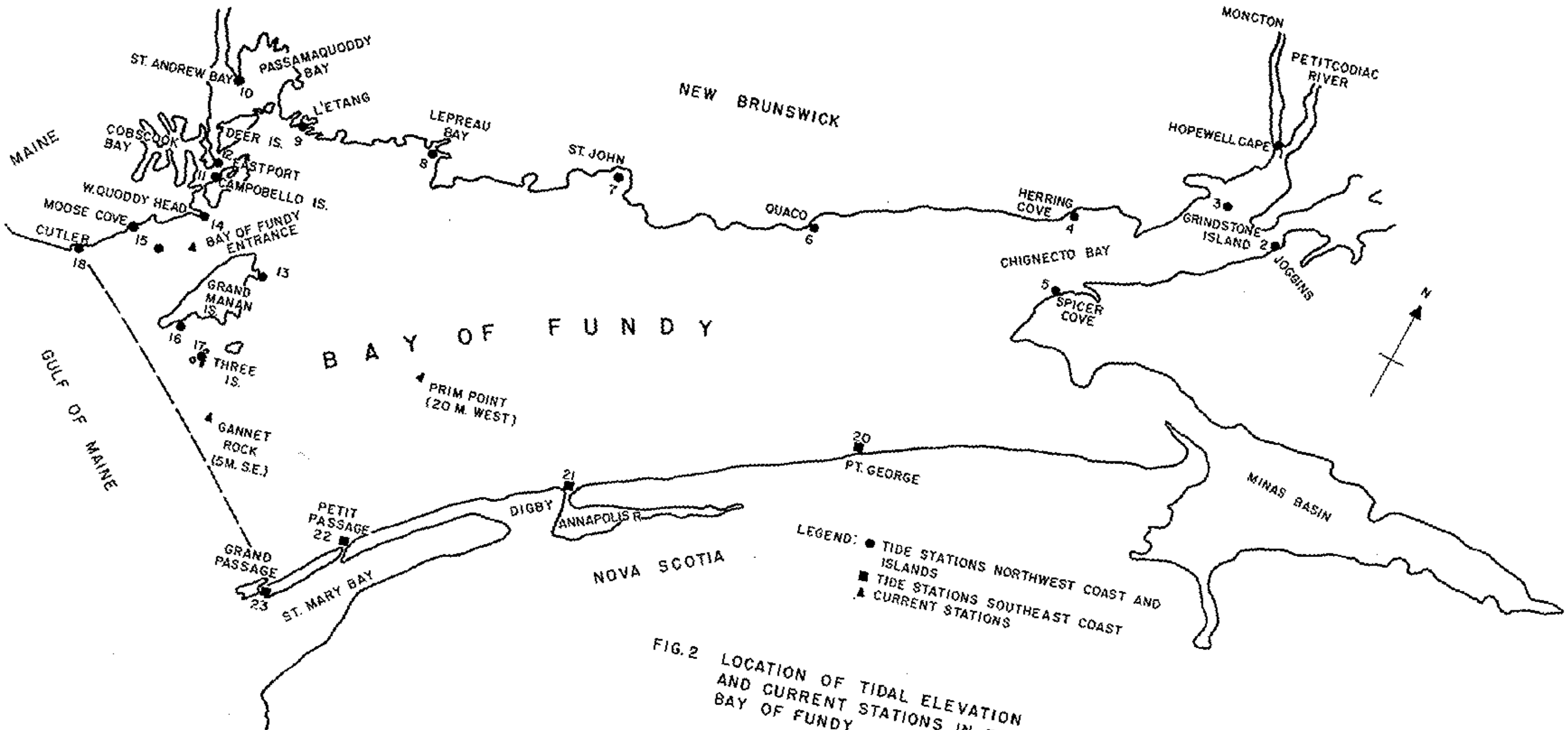

FIG. 2 LOCATION OF TIDAL ELEVATION AND CURRENT STATIONS IN THE BAY OF FUNDY

with observed values. This phase of the investigation is concerned with determining the degree to which the existing tides approach a condition of resonance within the basin.

2. To investigate the influence of flow disturbances on the general tidal elevations in the vicinity of the power development. The disturbances are to be analogous to the prospective effects of the Passamaquoddy system.

3. To draw conclusions leading either to the viewpoint that the effects of such disturbances are small or conversely that either more detailed calculations or experimental investigations are indicated.

II. REVIEW OF TIDAL THEORIES

The purpose of the following section is to review the theories of tidal motions in estuaries and embayments for the purpose of choosing the analytical approach best suited for the Bay of Fundy tidal system.

The tide in a basin with one end closed and the other end in communication with the ocean is known as a cooscillating tide. Such tides have something in common with seiches which are free oscillations whose periods depend only upon the geometrical shape of a completely closed basin. The basic difference between a cooscillating tide and a seiche is that the period of the cooscillating tide must coincide with that of the ocean forcing tide and is therefore independent of the basin geometry. If friction is neglected, both seiches and cooscillating tides may be considered as standing waves and it therefore follows that extreme water levels occur simultaneously over the entire basin. The standing wave in a basin having a length L less than one-quarter of the length of the standing wave, λ , is shown in Figure 3.

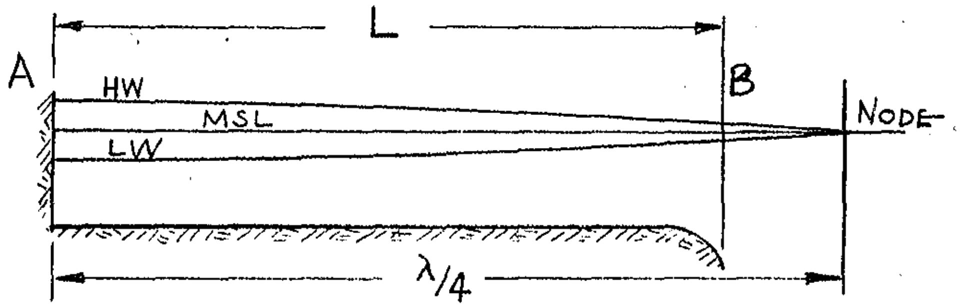

Figure 3. Standing Wave in a basin where L $<$ $\lambda/4$.

In this case the range of the ocean tide at the entrance is shown at B and the greater range at the head of the estuary at A. The basic equations of the frictionless cooscillating tide are given in several reference works, notably Lamb (1) and Proudman (2) (p. 230). Three cases have been solved by Lamb:

(a) Depth and width const.
(b) Depth constant, width proportional to length
(c) Width constant, depth proportional to length

Taylor (3) has extended the analysis to the case in which both depth and width are linearly proportional to length (and decrease to zero at the head of the estuary). This treatment was successfully applied by Taylor to predicting the tides in the Bristol Channel. Recently, Evangelisti (4) has considered the case of non-linear variation of depth and width.

Practically all of the prior treatments of the Bay of Fundy tides [i.e. Proudman (2) (p. 232), Defant (5) and Marmer (6) (p. 222)] show, by assuming a constant mean depth and width, that the period of the ocean (forcing) tide closely corresponds to the free period of oscillation of the Bay. The two periods become identical if $L = \lambda/4$ (Figure 3). Thus, the tides at A become larger as the node approaches point B, and the system nears a state of resonance. According to Sverdrup, et al (7) an exact computation of the period of free oscillation of the Bay of Fundy, taking account of the variation in depth and width, has not been undertaken.

There seems little justification for an attempt to refine the analysis of the Fundy tides from the standpoint of the frictionless standing wave. Fundamentally, the standing wave theory is incapable of predicting tidal amplitudes or velocities in a system near resonance since the analytical values approach infinity at resonance. It is therefore concluded that only additional qualitative information on the near resonance character of the Bay of Fundy would result from a consideration of the depth and width variation. This is due to the fact that in all such cases the character of the forced oscillation depends upon the relation between the forcing period and the free period.

In parallel with the theory of mechanical vibrations the analyses of near-resonant systems must consider the introduction of damping effects in the system. In a uniform rectangular basin in contact with an ocean at one end, friction provides the necessary damping effect which will prevent the development of pure resonance. In the absence of friction, the cooscillating tide can be considered as a wave of constant amplitude which proceeds up the basin and is totally relfected at the head of the bay. In the presence of friction, the amplitude of the incoming wave decreases in the direction of motion, the reflected wave therefore has a smaller amplitude and also undergoes damping in its progress toward the ocean. In the limiting case in which the incoming wave is completely dissipated by friction, the amplitude at the head of the basin and hence that of the reflected wave is zero. In this case the cooscillating tide reduces to a single progressive wave subject to extreme damping.

The damped cooscillating tide results in phase changes along the estuary so that the time of high or low water is a variable along the length of the estuary. The basic theory of the cooscillating tide is based on the classical work of Defant (5). Fjeldstad (8) considered the motion of a free wave undergoing damping as it advanced upon a continental coast from which it was reflected. Einstein and Fuchs (9) have used a similar approach to predict tidal amplitudes and velocities in a uniform rectangular basin, the hydraulic roughness of which is a known constant. Proudman (2) (p. 323) has recalculated the Bristol Channel tides taking friction into consideration and accounting for section changes by step-wise application of the differential equations of motion. This method also requires knowledge of a coefficient of frictional resistance for the channel and in addition it assumes that tidal velocities near the head of the estuary are known. Other step-wise procedures for tidal computations have been outlined by Wilber (10) and Dronkers and Schonfeld (11). In all cases, bottom frictional coefficients and tidal velocities or elevations must be known boundary conditions.

Redfield (12) has applied methods similar to those used by Proudman and Fjeldstad in the study of near resonant tidal systems including the Bay of Fundy. The method of damped cooscillating tides can be approximately applied to a real estuary of arbitrary geometry if the distribution of tidal ranges along the estuary is known. Inasmuch as this information is available in published tide tables, this type of analysis does not depend on assumed frictional resistance coefficients or assumed tidal velocities. The primary advantage of the method is that it eliminates the need for step-wise treatment of the estuary and it therefore defines the limits placed by damping on the amplification of the tidal wave by resonance. A detailed discussion of the mathematical model based on the foregoing considerations is given in the following section.

III. MATHEMATICAL MODEL OF THE BAY OF FUNDY

The reason for the development of a mathematical model for the Bay of Fundy is twofold.

First, it is desired to determine the "degree of resonance" inherent in the tidal system as a means of estimating the relative "sensitivity" of the system to disturbances.

Second, it is desirable to be able to compute the time variation and magnitude of the tidal velocities at a given station in the estuary as a means of estimating the relative importance of the flow disturbances induced by the tidal power project.

The theory of the damped cooscillating tide in a uniform basin will be presented initially. This will be followed by a discussion of the assumptions and procedure for applying the results to a non-uniform basin such as the Bay of Fundy.

Figure 4 is a definition sketch showing the notation to be used in the analysis.

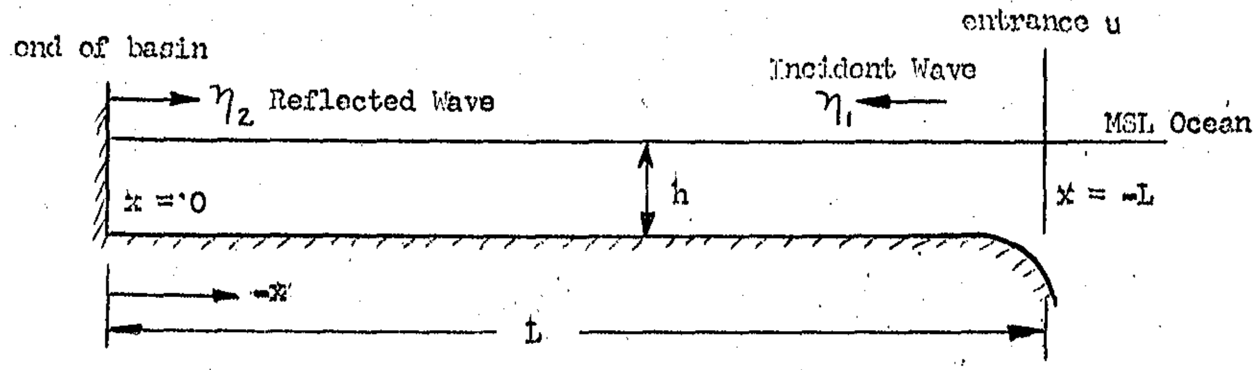

Figure 4. Definition sketch for a damped cooscillating tide in a uniform basin.

A. Tidal Elevations

The tide in the basin is assumed to be composed of two waves travelling in opposite directions: (1) an incident wave entering from the ocean (travelling in the positive x direction) having an amplitude η_1 which is a function of x and t (time). (2) a reflected wave (travelling in the negative x direction) having an amplitude η_2 also a function of x and t. If it is assumed that the amount of energy dissipated is always proportional to the total energy of the wave, the introduction of friction leads to a logarithmic decrease of the amplitude. The amplitudes of the incident and reflected waves are then given by the following equations:

$$\eta_1 = A\, e^{-\mu x} \cos(\sigma t - Kx) \qquad [1]$$

$$\eta_2 = A\, e^{\mu x} \cos(\sigma t + Kx) \qquad [2]$$

where $\sigma = \frac{2\pi}{T}$ (T = period of ocean tide = 12.42 hrs.)

$K = 2\pi/\lambda$ (λ = wave length)

A = amplitude of wave at end of basin (x = 0)

μ = damping coefficient

t = 0 corresponds to high water at end of basin

e = 2.718...

The tidal elevation, η, (above or below MSL) at any time, t, and place, x, in the basin is given by the sum of the amplitudes of the incident and reflected waves, thus,

$$\eta = \eta_1 + \eta_2 = A[e^{-\mu x} \cos(\sigma t - Kx) + e^{\mu x} \cos(\sigma t + Kx)] \qquad [3]$$

The time of high water at any position in the estuary occurs when $\partial\eta/\partial t = 0$ therefore,

$$\frac{\partial \eta}{\partial t} = e^{-\mu x} \sin(\sigma t - Kx) + e^{\mu x} \sin(\sigma t + Kx) = 0 \qquad [4]$$

Equation [4] may be expanded and regrouped in the following form:

$$(e^{\mu x} + e^{-\mu x}) \sin\sigma t \cos Kx + (e^{\mu x} - e^{-\mu x}) \cos\sigma t \sin Kx = 0$$

and since, $\dfrac{e^{\mu x} - e^{-\mu x}}{e^{\mu x} + e^{-\mu x}} = \tanh \mu x$

$$\tan\sigma t_H = - \tan Kx \tanh \mu x \qquad [5]$$

σt_H is the relative time of high water at any position in the estuary with respect to the time of high water at the end of the basin ($\sigma t_H = 0$).

Since, $\sigma = \dfrac{2\pi}{T} = \dfrac{360°}{T}$, and $\sigma t_H = \dfrac{360° \; t_H}{T} = \dfrac{360°}{12.42} t_H = (29°/\text{hr})\; t_H$

σt_H, will be expressed in angular measure. However, it may be converted to time measure, t_H in hours, by dividing the value of σt_H in degreees by ~~20°~~ 29°/hr.

From equation [5] the value of σt_H, in degrees, is given by,

$$\sigma t_H = \tan^{-1} (-\tan Kx \tanh \mu x) \qquad [6]$$

In order to determine the damping coefficient μ from published tidal data, it is convenient to express the local high water tidal amplitude, η_H in terms of the high water amplitude at the end of basin, η_{HO}.

The local high water amplitude at any point along the basin can be obtained from equation [3] by substituting the time angle for high water

σt_H into the equation. Equation [3] may be expanded and regrouped in the following form:

$$\eta = A[\cos\sigma t \ \cos Kx \ (e^{\mu x} + e^{-\mu x}) - \sin\sigma t \ \sin Kx \ (e^{\mu x} - e^{-\mu x}) \qquad [7]$$

and since,

$$1/2 \ (e^{\mu x} - e^{-\mu x}) = \sinh \mu x$$

$$1/2 \ (e^{\mu x} + e^{-\mu x}) = \cosh \mu x$$

equation [7] becomes,

$$\eta = 2A[\cos\sigma t \cos Kx \cosh\mu x - \sin\sigma t \sin Kx \sinh \mu x] \qquad [8]$$

By making use of the identity

$$\cos\sigma t = \frac{1}{\sqrt{1 + \tan^2\sigma t}} \qquad [9]$$

and the value of, $\tan\sigma t_H$, from equation [5] we have,

$$\cos\sigma t_H = \frac{1}{\sqrt{1 + \tan^2 Kx \tanh^2 \mu x}} \qquad [10]$$

Since, $\sin\sigma t_H = \tan\sigma t_H(\cos\sigma t_H)$, a similar operation yields,

$$\sin\sigma t_H = \frac{-\tan Kx \tanh \mu x}{\sqrt{1 + \tan^2 Kx \tanh^2 \mu x}} \qquad [11]$$

Equations [10] and [11] may now be substituted into the general equation [8] to obtain the local amplitude at high water, η_H. After a considerable process of trigonometric conversion and simplification, the local high water amplitude is expressed finally as:

$$\eta_H = 2A\sqrt{1/2 \ (\cos 2 Kx + \cosh 2\mu x)} \qquad [12]$$

at the end of basin where $x = 0$, $\cos (0) = \cosh (0) = 1$ therefore, $\eta_H = 2A = \eta_{oH}$ and the desired ratio of local and basin-end high water is as follows:

$$\left(\frac{\eta}{\eta_o}\right)_H = \sqrt{1/2 \ (\cos 2 Kx + \cosh 2\mu x)} \qquad [13]$$

To summarize, two basic equations have been determined: equation [6] gives the local time of high water relative to the basin end as a function of Kx and μx and equation [13] gives the ratio of local high tide relative to the basin end as a function of Kx and μx. Tidal data for given basins are available in the form of records of local time of high water and amplitude of high (and low) water. Thus, the theoretical equations have been placed in a form which makes possible their application to a real tidal system. In the analysis of an ideal uniform basin K is a constant and the damping coefficient μ can be determined directly by fitting equation [13] to a plot of $(\eta/\eta_o)_H$ versus Kx. The fact that K is a constant in a uniform depth basin can be readily shown by noting that long (tidal) waves in shallow water have a velocity of progress $C = \sqrt{gh}$ where h is the depth.

Since $C \equiv \lambda/T$

and $K = 2\pi/\lambda = 2\pi/CT = 2\pi/\sqrt{gh}\,T$ [14]

K is therefore constant since h and T are also constants.

In a non-uniform basin, h, is a variable and K would be expected to vary along the basin length. For any local station along the non-uniform basin both Kx and μx are unknowns. However, by making use of both equations [6] and [13] $(\eta/\eta_o)_H$ can be plotted against σt_H for arbitrary values of Kx and μx. Finally, it is assumed that in a channel of varying section the damping of the wave, μx, is directly proportional to the phase change along the channel Kx rather than to the distance travelled as in a uniform basin. This assumption may be expressed algebriacally in the following form:

$$\mu x = (\beta/2\pi)\ Kx \qquad [15]$$

where β is a constant of proportionality. Physically, the above equation means that the damping coefficient μ varies directly with K. K, in turn, varies inversely with the celerity of the wave and as the square root of the depth as shown by equation [14]. As the depth decreases both K and μ increase.

The validity of the assumed relation between μ and K can only be determined for a real basin by plotting the observed values of $(\eta/\eta_o)_H$ against σt_H and observing whether β is a constant for the entire basin.

B. Tidal Velocities

The equation for the tidal velocity can be developed for the uniform basin in accordance with the assumptions already used in the derivation of the tidal amplitude relations.

The continuity equation for unsteady non-uniform flow in an open channel considering amplitudes small compared to depth is given by:

$$\frac{\partial u_1}{\partial x} = -\frac{1}{h} \cdot \frac{\partial \eta_1}{\partial t} \qquad [16]$$

where u_1 is the velocity due to the incident wave of amplitude η_1.

With η_1 given by equation [1],

$$\frac{\partial \eta_1}{\partial t} = -A e^{-\mu x} \sigma \sin (\sigma t - Kx)$$

the continuity equation [16] becomes:

$$\frac{\partial u_1}{\partial x} = \frac{A\sigma}{h} e^{-\mu x} \sin (\sigma t - Kx) \qquad [17]$$

By integrating with respect to x, u_1 is obtained,

$$u_1 = \frac{A\sigma}{h} \int e^{-\mu x} \sin (\sigma t - Kx)\, dx$$

and by expanding the quantity in brackets,

$$u_1 = \frac{A\sigma}{h} \left[\sin \sigma t \int e^{-\mu x} \cos Kx\, dx - \cos \sigma t \int e^{-\mu x} \sin Kx\, dx \right]$$

or

$$u_1 = \frac{A\sigma}{h} \left[\sin \sigma t \; e^{-\mu x} \left(\frac{-\mu \cos Kx + K \sin Kx}{\mu^2 + K^2} \right) - \cos \sigma t \; e^{-\mu x} \left(\frac{-\mu \sin Kx - K \cos Kx}{\mu^2 + K^2} \right) \right] + F(t) \qquad [18]$$

The integration constant F(t) may be evaluated by introducing the boundary condition that $u_1 = 0$ at $x = 0$ (end of basin) for all values of time, hence, $F(t) = 0$.

Equation [18] may be considerably simplified by the following operations:

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\mu^2 + K^2} \left[\sin \sigma t \,(-\mu \cos Kx + K \sin Kx) - \cos \sigma t \,(-\mu \sin Kx - K \cos Kx) \right]$$

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\mu^2+K^2} \Big[K(\sin Kx \sin \sigma t + \cos Kx \cos \sigma t)$$

$$+ \mu (\sin Kx \cos \sigma t - \cos Kx \sin \sigma t) \Big]$$

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\mu^2+K^2} \left[K \cos (\sigma t - Kx) + \mu \sin (Kx - \sigma t) \right]$$

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\sqrt{\mu^2+K^2}} \left[\frac{K \cos (\sigma t - Kx)}{\sqrt{\mu^2 + K^2}} + \frac{\mu \sin (Kx - \sigma t)}{\sqrt{\mu^2 + K^2}} \right] \qquad [19]$$

let $\cos\alpha = \frac{K}{\sqrt{\mu^2 + K^2}}$, $\sin\alpha = \frac{\mu}{\sqrt{\mu^2 + K^2}}$

then, $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \mu/K$

and therefore, $\alpha = \tan^{-1} (\mu/K)$

From equation [15], it follows that

$$\alpha = \tan^{-1} (\beta/2\pi) \qquad [20]$$

Returning again to equation [19],

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\sqrt{\mu^2 + K^2}} \left[\cos\alpha \cos (\sigma t - Kx) + \sin\alpha \sin(Kx - \sigma t) \right]$$

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\sqrt{\mu^2 + K^2}} [\cos (Kx - \sigma t - \alpha)]$$

however, since the cos is an even function,

$$u_1 = \frac{A\sigma}{h} \frac{e^{-\mu x}}{\sqrt{\mu^2 + K^2}} [\cos(\sigma t - Kx + \alpha)] \qquad [21]$$

By a similar procedure the continuity equation is written for the reflected wave, with the result:

$$u_2 = -\frac{A\sigma}{h} \frac{e^{\mu x}}{\sqrt{\mu^2 + K^2}} [\cos (\sigma t + Kx + \alpha)] \qquad [22]$$

The tidal velocity at any time and position along the basin is given by the sum of u_1 and u_2, thus,

$$u = \frac{A\sigma}{h} \frac{1}{\sqrt{\mu^2 + K^2}} \left[e^{-\mu x} \cos(\sigma t - Kx + \alpha) - e^{\mu x} \cos(\sigma t + Kx + \alpha) \right] \quad [23]$$

Further discussion regarding the use of the tidal velocity equation for non-uniform channels will be given later.

IV. APPLICATION OF MATHEMATICAL MODEL TO BAY OF FUNDY

A. Tidal Amplitudes.

The aim of this phase of the analysis is to determine the constant of proportionality β between the damping coefficient μ and K [eq. 15] for the Bay of Fundy. In accordance with the previous discussion this can be done by plotting observed values of the tidal amplitudes against the time angle σt_H.

Reflection is assumed to occur near Hopewell Cape at the head of the Chignecto Channel at the mouth of the Petitcodiac River on which a well known tidal bore exists.* The mean range of tides at Hopewell Cape is 35.0 ft., hence, $2\eta_{oH} = 35.0$ ft. Table I lists the tidal observation stations in order of their descent along the northwest (Sta. 1-18) and southeast (Sta. 20-23) coasts of the Bay of Fundy. The reference for the tidal data is the 1958 Tide Tables for the East Coast of North and South America (13).

Column 1 of Table I lists the station number used in this report. Column 2 gives the USGS station number. The mean range of tide is listed in column 3 and the ratio of the mean tide at any station to the mean tide at Hopewell Cape $(\eta/\eta_o)_H$ is found in column 4.

The time of high water relative to the time of high water at the end of basin (Hopewell Cape) cannot be obtained directly from the tide tables. The procedure for the determination of σt_H is discussed below.

B. Time of High Water.

The vast majority of tidal stations listed in the tide tables are known as subordinate stations for which the time of high water is listed as a certain number of minutes before or after the time of high water at a primary station. Full daily predictions of time and height of high tides are given for the primary stations. It is noted that all of the subordinate stations used in this report are based on either St. John, New Brunswick, or Eastport, Maine. To obtain the times of high water

*The bore exceeds five feet on the highest spring tides.

relative to that at Hopewell Cape use is made of the high water lunitidal interval. The high water lunitidal interval is defined as the average time interval in mean solar hours between the passage of the moon across the meridan of the place and the next high water at the given place. In recent years the USGS has discontinued publication of the high water lunitidal intervals in the tide tables, however, reference may be made to tide tables for the year 1950 or earlier for determination of these tidal constants. The difference between the lunitidal intervals at two tidal stations, corrected for the difference in the longitudes of the two stations, gives the average difference between the time of high water at these stations. Let I_x and I_o be the lunitidal intervals for an arbitrary station and Hopewell Cape respectively, ℓ_x and ℓ_o their longitudes in degrees west of Greenwich, and t_H and t_{Ho} the standard time, in hours, of high water at the two stations. Then, since $t_{Ho} = 0$

$$t_H = I_x - I_o + .07\,(\ell_x - \ell_o)$$ [see Pillsbury (14), p. 5]

and $\sigma t_H = (29°/\text{hr.})\ t_H$

Columns 5, 6, 7, 8 and 9 in Table I list the details of the computation of σt_H for each station.

For certain of the subordinate stations (for which lunitidal intervals were not avilable) an alternative method of obtaining σt_H was used. Once the values of t_H for a primary station such as St. John or Eastport has been found, the value of t_H for any subordinate station can be obtained by adding or subtracting the time difference values tabulated in the present-day tide tables. For example, the value of t_H for St. John (Sta. No. 7) is $t_H = -.38$ hours, for Herring Cove (Sta. No. 4) the tide tables indicate a high tide 0.23 hours after high tide at St. John. Therefore, the value of t_H for Herring Cove is:

$$t_H = -.38 + .23 = -.15$$

The values of t_H for several stations were computed using lunitidal intervals and time differences and identical values of t_H were obtained. For those stations computed by the alternate method, the time difference and the reference station is indicated in Table I in place of the lunitidal interval.

C. Damping Constant for the Northwest Coast.

The tidal power development is situated along the northwest coast of the Bay of Fundy, therefore, attention will be directed initially toward an analysis of the stations along this side of the Bay.

TABLE I

BASIC TIDAL DATA FOR THE BAY OF FUNDY

A. Northwest Coast and Islands

(1) Sta. No.	and (2) C.&G.S. No.	(3) Mean Tidal Range	(4) Tidal Amplitude Ratio	(5) H. W. Lunitidal Interval	(6) Interval Diff.	(7) Longitude Diff.	(8) Time of High Water	(9) Phase of High Water	(10) Phase Change from end	(11) Phase Change from Ocean	(12) Tidal Amplitude Ratio
		$2\eta_H$	η_H/η_{Ho}	I	$I_x - I_o$	$l_x - l_o$	t_H	σt_H	K_x	$68^o + K_x$	η_H/η_{HL}
		ft.		hrs.	hrs.	deg.	hrs.	deg.	deg.	deg.	
1	587 Hopewell	35.0	1.00	11.76	0	0	0	0	0	68	2.50
2	581 Joggins	33.0	0.94		(+0.30 on St. John)		-.08	- 2	-20	48	2.35
3	585 Grindst.	32.0	0.91	11.70	-.06	+.05	-.06	- 2	-24	44	2.28
4	601 Herring	27.0	0.77		(+0.23 on St. John)		-.15	- 4	-40	28	1.925
5	579 Spicer	27.0	0.77		(+0.20 on St. John)		-.18	- 5	-40	28	1.925
6	603 Quaco	23.1	0.66	11.50	-.26	+.96	-.19	- 6	-49	19	1.65
7	605 St. John	20.6	0.59	11.28	-.48	+1.50	-.38	-11	-55	13	1.475
8	609 Lepreau	19.0	0.54	11.23	-.53	+1.91	-.40	-11.5	-58	10	1.35
9	611 L'Etang	18.4	0.525	11.25	-.51	+2.25	-.36	-10.5	-59	9	1.31
10	623 St. Andr.	19.2	0.55		(+.13 on Eastport)		-.34	-10	-57	11	1.375
11	621 Welshpl.	18.3	0.525		(+.02 on Eastport)		-.45	-13	-59.5	8.5	1.31
12	627 Eastport	18.2	0.52	11.12	-.64	+2.40	-.47	-13.5	-60	8	1.30
13	613 N. Gr. Man.	17.5	0.50		(-.20 on St. John)		-.58	-17	-62	6	1.25
14	651 W. Quoddy	15.7	0.45	10.97	-.79	+2.41	-.62	-18	-65	3	1.125
15	653 Moose C.	14.8	0.42	10.96	-.80	+2.53	-.62	-18	-66.5	1.5	1.05
16	615 S. Gr. Man	14.3	0.41		(-.25 on St. John)		-.63	-18	-67	1	1.025
17	617 Three Is.	14.4	0.41	10.82	-.94	+2.19	-.79	-23	-68	0	1.025
18	655 Cutler	13.9	0.40	10.93	-.83	+2.65	-.64	-18.5	-68	0	1.00

B. Southeast Coast

(1) Sta. No.	and (2) C.&G.S. No.	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
20	563 Pt. George	25.3	0.72		(-.10 on St. John)		-.48	-14	-48		
21	559 Digby	20.6	0.59		(-.30 on St. John)		-.68	-20	-58		
22	555 P. St. Mary	15.8	0.45		(-.60 on St. John)		-.98	-28.5	-67		
23	553 G. St. Mary	14.7	0.42		(-.52 on St. John)		-.90	-26	-68		

The tabulated values of $(\eta/\eta_o)_H$ and σt_H are plotted on a semi-logarithmic scale in Figure 5. Also plotted are the theoretical curves, Eqs. [6], [13] and [15], for constant values of the phase shift Kx and the damping proportionality parameter β for values 0.5, 0.8, 1.0 and 1.3. The observed values for the northwest side are designated by circles and are seen to agree closely with a constant value of $\beta = 0.8$ for stations as far south as Cutler, Maine, which is considered to be the junction between the Gulf of Maine and the Bay of Fundy. From Figure 5 the value of Kx for each of the tidal stations can be obtained by interpolation, these values are tabulated in column 10 of Table I.

D. Damping Constant for the Southeast Coast.

The tabulated values of the amplitude ratio and high water time angle for the southeast coast are plotted as squares in Fig. 5. There are only a few tidal stations on the outer shore of the southeast coast and the scatter of the points is somewhat greater than for the opposite shore. Nevertheless the curve for $\beta = 1.3$ may be considered as an average for the southeast coast of the Bay.

It is noted that the total phase shift Kx to the Gulf of Maine is about -68° for both sides of the Bay and that for equal phase shifts the tides on the southeast coast are roughly ten per cent higher than those of the northwest coast. It is of interest to see whether the change in the tidal amplitudes for the two sides can be explained by the effect of the earth's rotation.

E. Geostrophic Effects.

The effect of the earth's rotation upon the tidal cooscillation in a basin is usually known as the geostrophic or Coriolis' effect. The magnitude of the geostrophic effect at a given location along the axis of the basin can be shown quite easily if the tidal cooscillation is assumed to be frictionless in a uniform basin. For this case, (Proudman (2), p. 255) the equation for the tidal amplitude at any time and place in the basin is:

$$\eta = A\,[e^{-\gamma} \cos(\sigma t - Kx) + e^{\gamma} (\cos \sigma t + Kx)] \qquad [24]$$

where $\gamma = 2\pi\, y/T_p \sqrt{gh}$

and y is the horizontal width coordinate at right angles to the x axis of the basin.

T_p is a period of time known as the "half-pendulum day." At the poles T_p is equal to half a sidereal day or 12 hours. At any other latitude, $\emptyset$, T_p is given by

$$T_p = \pi / \omega \sin \emptyset$$

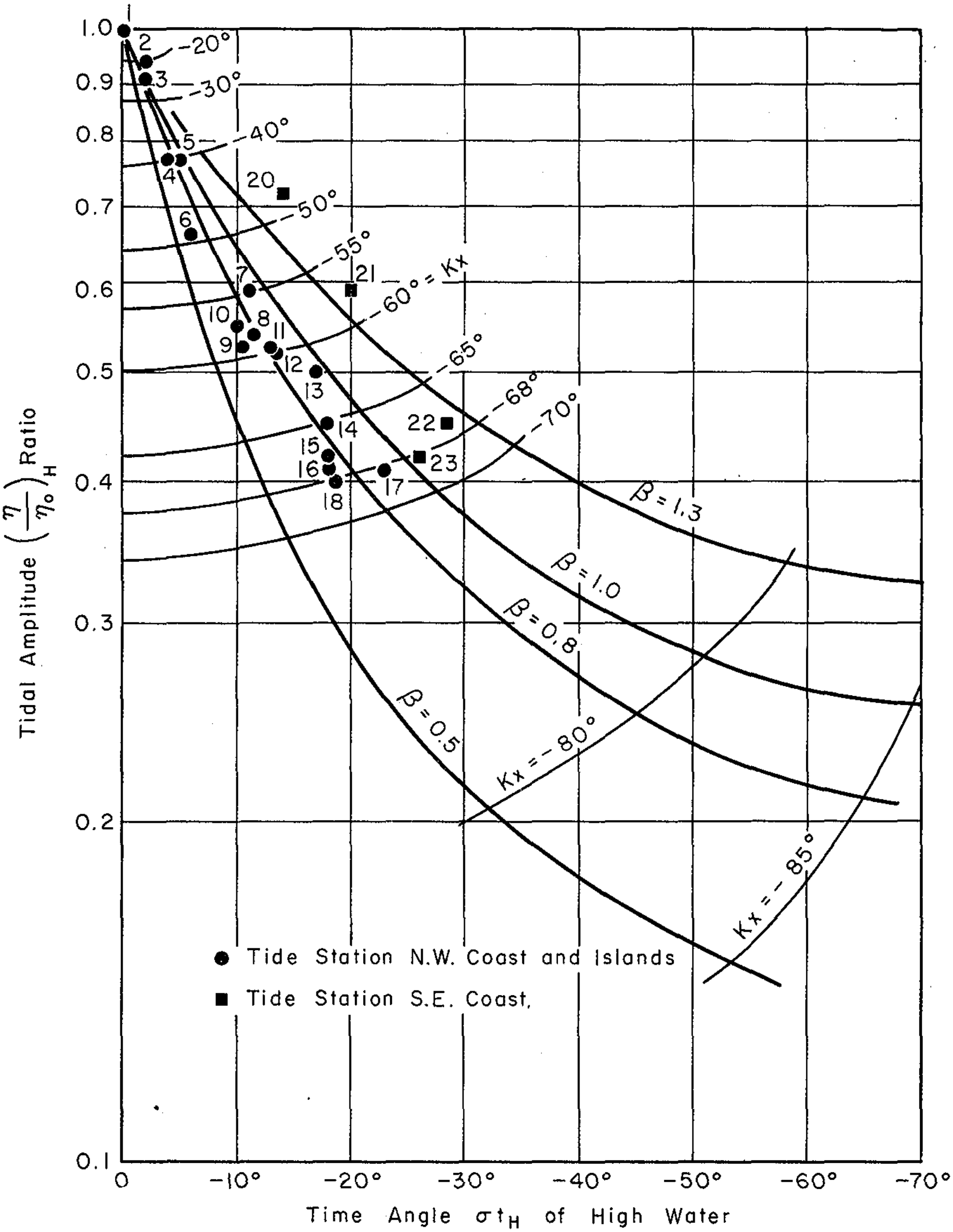

FIG. 5 DETERMINATION OF DAMPING PROPORTIONALITY FACTORS FOR THE BAY OF FUNDY

where ω = angular speed of the earth's rotation = 7.29×10^{-5} rad./sec.

hence $\gamma = 2\, y\, \omega \sin \phi \,/\, \sqrt{gh}$

The geostrophic effect is zero if we move down the centerline of the basin where $\gamma = 0$ (since $y = 0$). It will be assumed that the damped cooscillating tidal theory can be used to determine the basin centerline values and that equation [24] will describe the change in amplitude from one side of the basin to the other.

As an example, two tidal stations located essentially opposite one another and having the same phase shift, Kx will be chosen. Stations satisfying this requirement are Lepreau Bay, (No. 8), New Brunswick, and Digby, (No. 21), Nova Scotia. The width of the Bay of Fundy in this region is 220,000 ft.; hence, y = 110,000 ft., the mean depth, h = 270 ft. and the latitude, $\phi = 45°$, hence

$$\gamma = \frac{2(1.1 \times 10^{5})(7.29 \times 10^{-5})(.707)}{\sqrt{(32.2)\ 270}} = 0.12$$

From Figure 5 it is seen that a phase shift, $Kx = -58°$ is approximately correct for both Lepreau Bay and Digby. If it is assumed that $\beta = 1.05$ is the curve corresponding to the centerline of the bay a value of $\sigma t_H = -16°$ may be interpolated.

Equation 24 may now be applied to determine the ratio of tidal elevations between the northwest (Lepreau Bay) and southeast (Digby) coasts at this point in the basin. It is noted that the positive y axis is always to the left of the positive x axis, therefore +y corresponds to Lepreau Bay and -y to Digby.

Therefore,

$$\frac{\eta_{+y}}{\eta_{-y}} = \frac{e^{-\gamma} \cos(\sigma t - Kx) + e^{\gamma} \cos(\sigma t + Kx)}{e^{\gamma} \cos(\sigma t - Kx) + e^{-\gamma} \cos(\sigma t + Kx)}$$

with $Kx = -58°$

$\sigma t_H = -16°$

$\gamma = 0.12$

then

$$\frac{\eta_{+y}}{\eta_{-y}} = 1/1.12$$

The high tide at Digby is therefore computed to be 12% higher than the tide at Lepreau Bay. The observed values from Table I indicate that the actual tide at Digby (21.0 ft.) compared to that at Lepreau Bay (19.0 ft.) is 10.5% higher.

The foregoing analysis of the geostrophic effect is therefore accepted as the explanation for the different values of β obtained for the northwest and southeast coasts of the Bay of Fundy.

F. Tidal Velocities.

Information has been obtained to verify the assumption of a linear proportionality between the damping coefficient μ and the phase constant K [equation (15)]. It is now desirable to have a means of applying the tidal velocity equations previously developed for a uniform channel to the Bay of Fundy. This will permit the computation of the tidal velocity variation with time for any section along the axis of the Bay. From the foregoing analyses of the geostrophic effects the mean value of $\beta = 1.05$ will be used for velocity calculations pertaining to a section of the basin.

The general tidal velocity equation for a uniform basin is given in equation [23]. For convenience, the equation will be repeated below without writing out the quantities appearing within the brackets.

$$u = \frac{A\sigma}{h} \frac{1}{\sqrt{\mu^2 + K^2}} \left[- - - \right] \qquad [23]$$

From equation [15] and with $\beta = 1.05$

$$\frac{\mu}{K} = \frac{1.05}{2\pi} = \frac{1}{6} \qquad [25]$$

Equation [23] may be rewritten as:

$$u = \frac{A\sigma}{h} \frac{1/K}{\sqrt{(\mu/K)^2 + 1}} \left[- - - \right] \approx \frac{A\sigma}{Kh} \left[- - - \right] \qquad [26]$$

since, $(\mu/K)^2$ is small and $\sqrt{(\mu/K)^2 + 1} \approx 1$.

Making use of the previous notation, that

$$A = \frac{\eta_{oH}}{2}, \quad K = \frac{2\pi}{T\sqrt{gh}}, \quad \sigma = \frac{2\pi}{T} \text{ and } \alpha = \tan^{-1}(\mu/K) = \tan^{-1}(\tfrac{1}{6}) = 9\text{-}1/2°$$

the general tidal velocity equation [26] may be written in the form,

$$u = \frac{g\,\eta_{oH}}{2\sqrt{gh}}\Big[e^{-1.05\ Kx/2\pi}\ \cos(2\pi\tfrac{t}{T} - Kx + 9\text{-}1/2°)$$

$$- e^{1.05\ Kx/2\pi}\ \cos\,(2\pi\tfrac{t}{T} + Kx + 9\text{-}1/2°)\Big] \qquad [27]$$

Equation [27] may be applied with good approximation to the lower reaches of the Bay of Fundy where the change in depth with distance is relatively small. The tidal velocity at any instant of time is a function of the high water amplitude at end of basin, the depth and the value of Kx for the section of the basin under consideration.

G. Computation of Tidal Elevations and Velocities in the Vicinity of Eastport, Maine.

As a check on the validity of the mathematical model of the Bay of Fundy the equations which have been developed will be used to compute:

(1) the time variation in tidal elevation at Eastport

(2) the time variation in tidal velocities for the sections of the Bay of Fundy adjacent to Eastport, and Cutler, Maine.

The general tidal elevation equation for the northwest Coast can be obtained by substituting $A = \eta_{oH}/2$ and $\beta = 0.8$ $(\mu x = (0.8/2\pi)\ Kx)$ into equation [3],

$$\eta = \frac{\eta_{oH}}{2}\ \left[e^{-.8Kx/2\pi}\cos(2\pi\tfrac{t}{T} - Kx) + e^{.8Kx/2\pi}\cos(2\pi\tfrac{t}{T} + Kx)\right] \quad [28]$$

According to Figure 5, the value of Kx for Eastport (Sta. 12) is Kx = -60°. The mean depth of the Bay of Fundy at this section is h = 340 feet, and the high water amplitude at Hopewell Cape, η_{oH} = 17.5 feet. With the above information [eq. 28] and [eq. 27] can be calculated for various times t/T. For Cutler, Kx = -68° and the mean depth h = 325 feet.

The comparison of the computed tidal elevations with the observed values for a mean tide at Eastport is shown in Figure 6. The agreement in both range and phasing indicates that the mathematical model is capable of describing the tidal amplitude variations at a particular location.

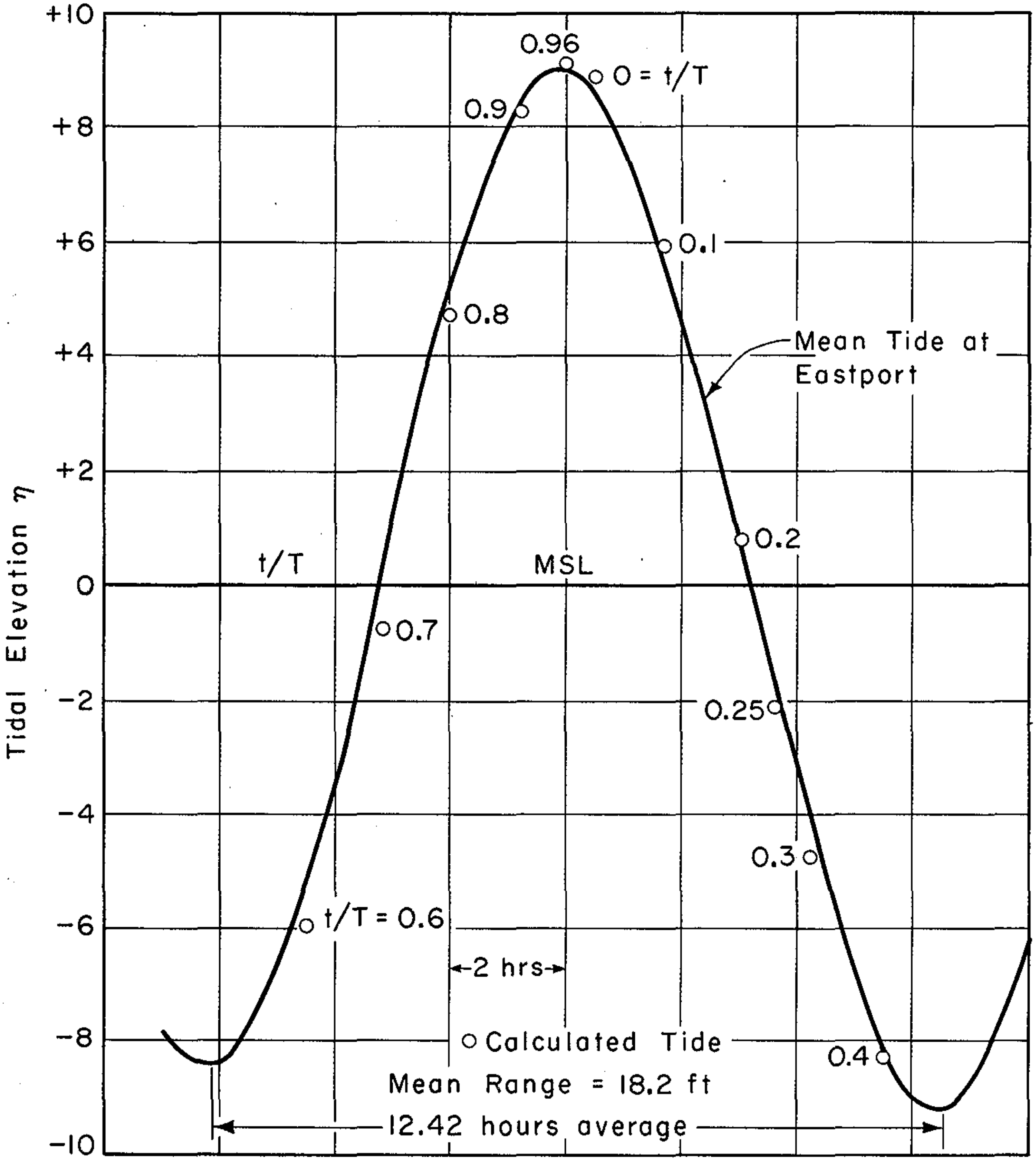

FIG. 6 – COMPARISON OF CALCULATED AND OBSERVED TIDE AT EASTPORT

The computed values of the tidal velocities for the Eastport section of the Bay of Fundy over a mean tidal cycle are given in Table II. The calculations indicate a maximum value of the flood or ebb velocity of 4.8 ft./sec. or 2.8 knots. Similar calculations for the section between Cutler, Maine and Grand Passage, St. Mary Bay, Nova Scotia, indicate a maximum flood or ebb velocity of 5.2 ft./sec. or 3.1 knots. A tidal current observation station located in the middle of the Bay at the Cutler-St. Mary section (5 miles SE of Gannet Rock, Ref. 15) gives a mean of the maximum flood or ebb velocities of 3.3 knots. Another station (Bay of Fundy entrance, Ref. 15) slightly northeast of the Cutler section gives an observation of 2.4 knots. Only one current observation station exists in the vicinity of the Eastport section (20 miles west of Prim Point, Ref. 15), the observed maximum current there is 1.5 knots. In view of the agreement of the calculated and observed velocities at the Cutler section, the agreement for the Eastport section is poor. This may be due in part to the fact that this observation station is located at the deepest portion of the section (420 feet) whereas the mean depth is 340 feet. Both observation stations at the Cutler section are in regions where the depth is essentially equal to the mean depth. Since the calculated and observed tidal elevations at Eastport are in good agreement (Fig. 6) it seems reasonable to expect that the calculated velocities would not be greatly in error. One further check on the validity of the tidal velocity calculations can be made by comparing the predicted time of maximum velocity with the observed value. For the current observation station 20 miles west of Prim Point (Eastport section) the tabulated (Ref. 15) lunitidal interval for the maximum flood velocity is 9.00 hrs. By making the appropriate longitude correction, it is determined that the strength of flood at the Eastport section occurs at $t/T = 0.78$. From Table II it is seen that the theoretical time of maximum flood velocity occurs at $t/T = 0.75$.

V. ANALYSIS OF EFFECTS OF PASSAMAQUODDY TIDAL POWER DEVELOPMENT

A. Introduction.

A rigorous mathematical treatment of the effect of superimposing a disturbance representing the power development on the existing tidal system would be highly desirable. Such a treatment would consist of placing a point source or sink, simulating the addition or subtraction of water through the project control gates, at a point on the side of the geometric model representing the basin. One would then look for the effect of the source or sink on the tidal amplitudes in the basin. This approach although mathematically complex, and as yet unsolved even for a rectangular basin, would be possible if the tidal system could be represented by an undamped cooscillating tide. An investigation of the possibility of conducting this type of analysis for the simple geometry described above has disclosed that if the Bay of Fundy is represented by an undamped tide in a uniform basin a condition of almost perfect resonance results. Under these conditions the small amplitude wave theory inherent in the superposition methods breaks down. The mathematical model which has been used in representing the tidal system indicates that the damping effect is important in that it removes the system from one of near perfect resonance to one of damped resonance in which rather modest tidal amplifications are obtained. This point will be discussed further in the succeeding analysis of the effects of the power development.

As a basis for evaluating the disturbance effects of the tidal power development in a somewhat less rigorous manner, several more or less independent lines of investigation are proposed. Taken in total, it is expected that they will provide a quantitative measure of the relative importance of the proposed power development on the amplitude of the tide. The proposed methods of analysis are considered in detail in the following sections.

B. Comparison of Controlled and Uncontrolled Flow Rates in Passamaquoddy and Cobscook Bays.

As a means of estimating the magnitude of the disturbance introduced by the proposed tidal power development, the pattern of inflow and outflow from Passamaquoddy and Cobscook Bays will be determined for the existing uncontrolled condition. By superimposing the proposed controlled flows on the existing flows, the magnitude of the flow change caused by the construction of the closure dikes can be determined. This comparison will be used in the subsequent analysis which is related to the mathematical model of the entire tidal system.

A curve showing the combined inflow or outflow rates versus time during one tidal cycle for Passamaquoddy and Cobscook Bays in the uncontrolled or existing condition can be prepared in the following manner.

Let: S = surface area of Bay (ft.^{2})

$2\eta_H$ = range of tide in Bay (LW to HW) (ft.)

Vol. = total volume change in Bay (LW to HW) (ft.^{3})

then $\text{Vol.} = 2\eta_H S$

and the average rate of inflow (or outflow) in cfs. during the half tidal period of 6.21 hours (22,360 sec.) is

$$Q_{Av.} = \frac{\text{Vol.}}{\text{time}} = \frac{2\eta_H S}{22{,}360} \qquad \text{(cfs.)}$$

Since the tidal elevation and velocities vary sinusoidally, the discharge versus time is also sinusoidal and the peak rate of flow is $\pi/2$ times the average rate, hence,

$$Q_{max} = \pi/2 \, Q_{av} = \frac{\eta_H S}{7,120} \quad \text{(cfs)}$$

With the data given below for Passamaquoddy and Cobscook Bay the value of Q_{max} can be obtained for each Bay.

Passamaquoddy Bay		Cobscook Bay	
S	$= 2,640 \times 10^6$ ft.^{2}	S	$= 1,500 \times 10^6$ ft.^{2}
η_H	$= 9.6$ ft.	η_H	$= 9.4$ ft.
$Q_{max.}$	$= 3.56 \times 10^6$ cfs.	$Q_{max.}$	$= 1.98 \times 10^6$ cfs.

The tide tables show that there is essentially no phase lag between the time of high tide at Eastport and the time of high tide in the two adjacent Bays. Therefore, the curves of discharge for these bays are plotted as dashed lines in Fig. 7 against the local tidal time for Eastport which is designated t_E. The time, $t_E = 0$, corresponds to high tide at Eastport to distinguish it from the previous time system based on high tide at Hopewell Cape ($t = 0$). The sum of the discharge curves for the two bays is the total uncontrolled flow in or out of the combined Passamaquoddy and Cobscook Bays.

Figure 7 also shows the controlled inflow and outflow from the combined area from data furnished by the Corps of Engineers. The plan for operation of the power development indicates that inflow to Passamaquoddy Bay (upper pool) begins two hours before mean high tide at Eastport and ends 0.6 hours after mean high tide. The outflow from Cobscook Bay (lower pool) begins 3.55 hours after MHW and ends 7.15 hours after MHW at Eastport.

In the uncontrolled state, the peak rate of inflow or outflow from the two bay system is 5.54×10^6 cfs. In the controlled condition the peak rate of inflow is 1.75×10^6 cfs and 1.35×10^6 cfs for the outflow. If the lateral inflow or outflow for the two bay region is viewed as a type of "loading" on the longitudinal tidal system of the Bay of Fundy, it is concluded that the power development controls represent a decrease in the system loading. The possible effects of such changes in the system load will be discussed in relation to the mathematical model in a later section.

C. Comparison of Lateral Flows in Passamaquoddy Area with Longitudinal Flows in Bay of Fundy.

In the preceding section, the changes in the flow rates in the combined two bay area due to the power development were given. A further insight into the effect of such changes on the entire tidal system can

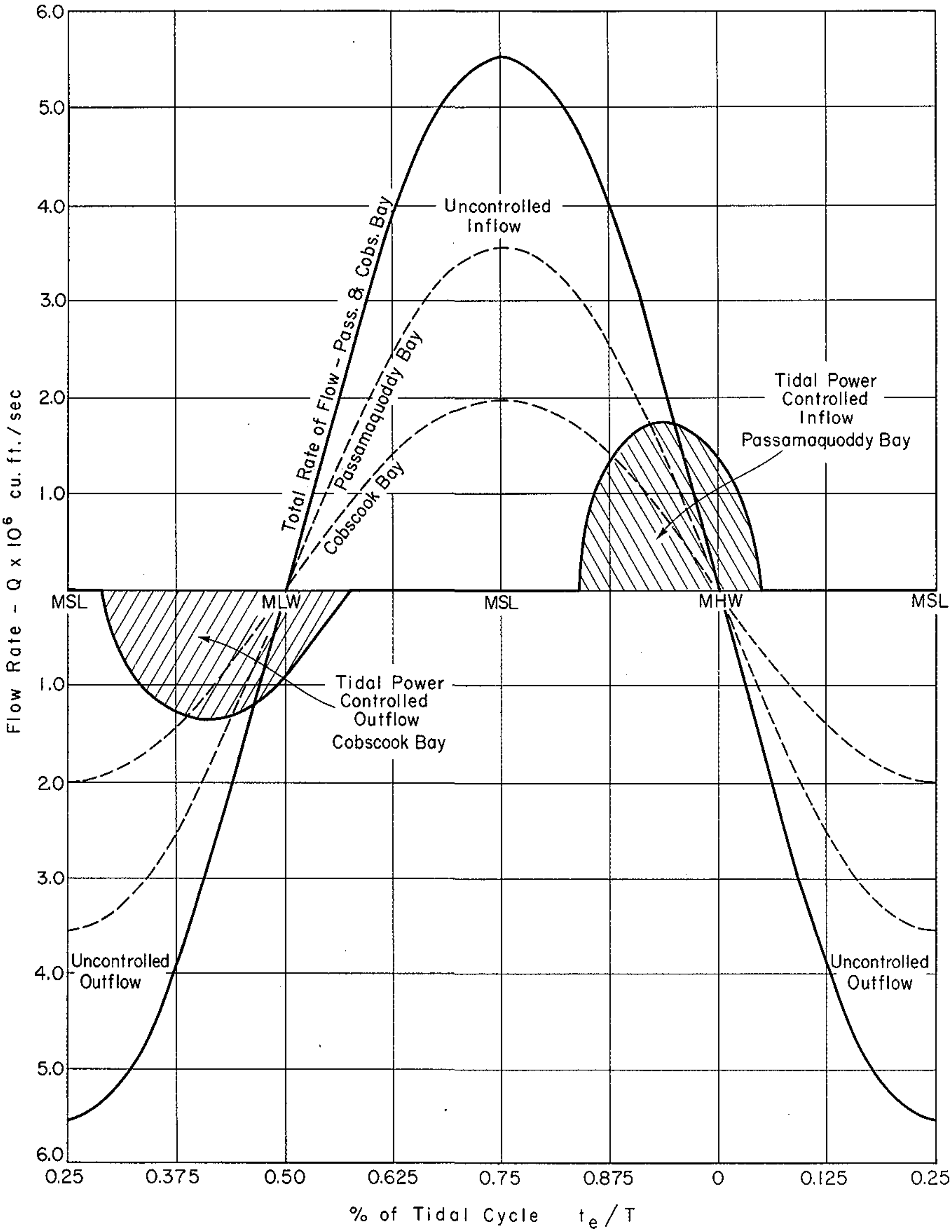

FIG. 7 – COMPARISON OF FLOW RATES IN PASSAMAQUODDY AND COBSCOOK BAYS BEFORE AND AFTER THE TIDAL POWER DEVELOPMENT.

be gained by comparing the magnitudes of these lateral (right angles to the basin axis) discharges with the longitudinal tidal discharge of the entire section of the Bay of Fundy adjacent to Eastport.

The tidal discharge versus time for the Bay of Funday section can be obtained by multiplying the tidal velocities of Table II by the cross-sectional area of the Bay at the Eastport section. The resulting values of the tidal discharge are also tabulated in Table II. The cross-sectional area is given by the product of the mean depth, h = 340 feet, and the width of 280,000 ft., giving an area of 95×10^6 ft.^{2}. The longitudinal tidal flow rate of the Eastport section of the Bay of Fundy reaches a maximum of 450×10^6 cfs. on both the flood and ebb tide. A graphical superposition of the controlled or uncontrolled flows for the Passamaquoddy-Cobscook Bays (Fig. 7) with the tidal flow in the Bay of Fundy is not very helpful since the latter flow rates are of the order of one hundred times greater. The following percentagewise comparisons are therefore more meaningful:

1. Existing Condition (uncontrolled)

   The maximum lateral inflow rate for Passamaquoddy-Cobscook Bays is 5.54×10^6 cfs at $t_E/T = .75$. At the same instant of time the longitudinal tidal flow rate in the Bay of Fundy is 443×10^6 cfs. The corresponding ratio of uncontrolled lateral to longitudinal flow is 1.25 per cent. The same ratio holds for both inflow and outflow conditions.

2. Controlled Condition

   The ratio of controlled inflow peak rate (1.75×10^6) at $t_E/T = .94$ to the Bay of Fundy inflow at the same instant of time (240×10^6) is 0.75 per cent. The ratio of controlled outflow peak rate (1.35×10^6) at $t_E/T = .41$ to the Bay of Fundy outflow at the same instant (303×10^6) is 0.45 per cent.

In terms of the ratio of the peak lateral discharge to the longitudinal flow rate, the effect of the power development is to change the inflow rate from 1.25% to 0.75% and the outflow rate from 1.25% to 0.45%.

D. Sensitivity of the Damped Cooscillating Tide.

The damped-resonance mathematical model of the Bay of Fundy gives the tidal amplitude at any position along the basin in terms of an amplification factor. The amplification factor expresses the ratio of the local tidal amplitude, η_H, to the tidal amplitude existing at the ocean entrance, η_{HL}, and will be shown to depend upon the magnitude of the damping proportionality factor β. Estimates of the sensitivity of the local tidal amplitude for the northwest coast of the Bay of Fundy to changes in the damping proportionality factor may then be made.

Figure 8 has been prepared by plotting, for several values of β, the tidal amplification factor as the ordinate against the phase change relative to the ocean entrance as an abscissa. The following previously determined constants are used in the computation of the theoretical curves in Fig. 8.

1. Phase change of ocean entrance (Cutler, Maine) relative to end of basin (Hopewell Cap, N.B.)
   $Kx_L = -68°$ (see Fig. 5)

2. $2\eta_{HO}$ (Hopewell Cape) = 35.0 ft.

   $2\eta_{HL}$ (Cutler, Maine) = 13.9 ft.

$$\text{hence, } \frac{\eta_{HO}}{\eta_{HL}} = 2.5 \qquad [29]$$

Substituting eq. [15] into eq. 13 and multiplying by the ratio $\frac{\eta_{HO}}{\eta_{HL}}$

the equation for the tidal amplification factor becomes,

$$\frac{\eta_H}{\eta_{HL}} = 2.5 \sqrt{1/2 \left(\cos 2Kx + \cosh 2\beta \cdot Kx/2\pi\right)} \qquad [30]$$

where Kx varies between 0° and -68°.

The phase change relative to the ocean entrance is given by $(68° + Kx)$ since this quantity becomes zero for Cutler where $Kx = -68°$.

The observed tidal information may also be plotted on Fig. 8 by multiplying the tidal ratios, η_H / η_{HO}, in column (4) of Table I by equation [29]. The results are given in columns (11) and (12) of Table I. The observed tidal data for the northwest coast are again shown to fall along the curve corresponding to $\beta = 0.8$. In addition to this curve, the theoretical curves for $\beta = 0.5$ and $\beta = 1.0$ are shown in Fig. 8.

Possible changes in the tide due to the tidal power development in the vicinity of Eastport are the primary interest of this investigation. From Fig. 8, it is seen that the tidal amplification factor for Eastport is 1.30 and that the amplification is relatively insensitive to changes in the value of β. For example, if β changes from 0.8 to 1.0, an increase of 20%, the tidal amplification factor for Eastport changes to 1.26 which represents a 3% decrease in the amplitude of high tide at Eastport. If the value of β changes from 0.8 to 0.5, a decrease of 37%, the amplification factor becomes 1.31 or an increase of 0.8% in the tide at Eastport.

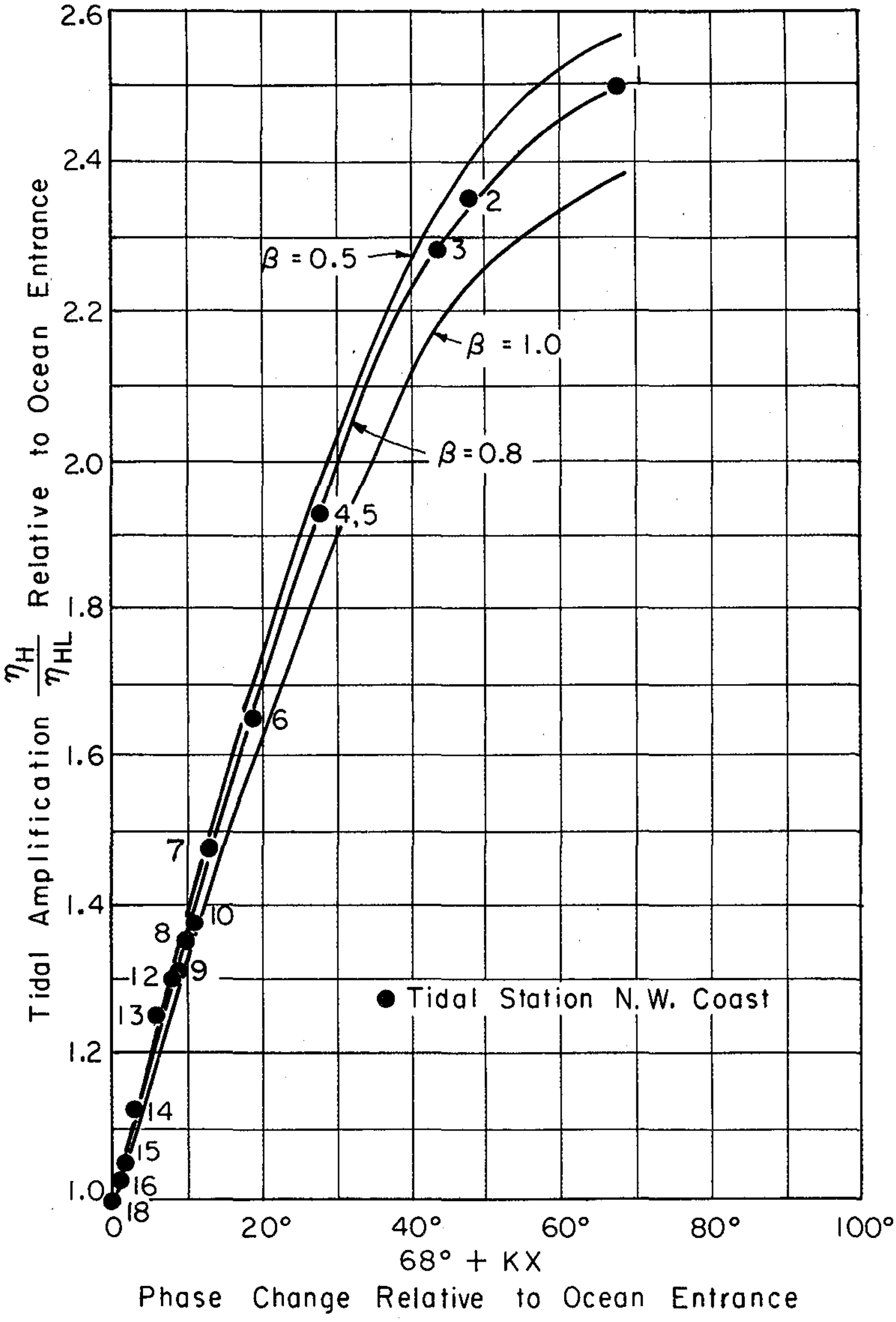

FIG. 8 TIDAL AMPLIFICATION VERSUS PHASE CHANGE FROM BAY OF FUNDY ENTRANCE

From the analyses of the preceding sections, it was observed that the construction of the tidal power controls will represent a decrease in the lateral flows in and out of the Bay of Fundy. It is therefore of interest to determine whether this change in the lateral flows would tend to increase or decrease the value of β.

The effect of lateral flows into and out of the Passamaquoddy-Cobscook Bay area have been previously referred to as a loading on the longitudinal oscillations of the Bay of Fundy. Use can be made of an analogy with the differential surge tank to relate the lateral flows with the flow through the port of the differential tank. The oscillations of the Bay of Fundy proper are therefore related to the oscillations in the riser of the differential tank. If the port area is zero the differential tank becomes identical with a simple surge tank which is a system of minimum damping. As the port area and therefore the ratio of lateral to longitudinal flow is increased the damping is also increased. By analogy, a reduction of the lateral flows caused by the tidal power development would be expected to decrease rather than increase the proportionality factor β. As is shown in Figure 8, a decrease in β will cause a very slight increase in the local tidal amplitudes.

TABLE II

Computed Tidal Velocities and Discharges
For the Section of the
Bay of Fundy Adjacent to Eastport (Eq. 27)

Percent of Tidal Cycle (t_E = 0, H.W. at Eastport) t_E/T	Percent of Tidal Cycle (t = 0, H.W. at Hopewell) t/T	Tidal Velocity ft/sec	Tidal Velocity u Knots	Tidal Discharge c.f.s. $\times 10^6$
0	.96	+.89	+0.5	+84
.04	0	-.32	-0.2	-30
.14	.10	-3.08	-1.8	-290
.24	.20	-4.62	-2.7	-440
.29	.25	-4.78	-2.8	-450
.34	.30	-4.43	-2.6	-420
.44	.40	-2.54	-1.5	-240
.54	.50	+0.32	+0.2	+30
.64	.60	+3.08	+1.8	+290
.74	.70	+4.62	+2.7	+440
.79	.75	+4.78	+2.8	+450
.84	.80	+4.43	+2.6	+420
.94	.90	+2.54	+1.5	+240

Note: plus signs indicate inflow or flood tide.
minus signs indicate outflow or ebbtide.

VI CONCLUSIONS AND RECOMMENDATIONS

1. A reasonably accurate mathematic model of the existing Bay of Fundy tidal system has been achieved. The analytical method indicates that the tidal system can be treated as a damped cooscillating tide. A constant of proportionality relating the damping coefficient to the phase change along the channel is shown to describe the combined effects of friction, depth and width changes and shoreline irregularities. Geostropic effects due to the earth's rotation are shown to account for tidal elevation differences between the northwest and southeast coasts of the Bay of Fundy.

2. The most general conclusion of the analytical study is the fact that the tidal system of the Bay of Fundy is not highly resonant. The usual example, cited in the literature, of the Fundy tides as near resonant comes about by assuming that the tides can be represented by an undamped cooscillating (standing-wave) tide in a basin of constant mean depth. Under these assumptions a total phase shift of 83° is obtained whereas a phase shift of 90° would indicate perfect resonance and infinite amplitudes at the head of the basin. In the damped cooscillating system representing the Bay of Fundy, the phase change is only 68° and the amplification of tide from the ocean entrance to the head of the Bay is two and one-half. An analysis of the effect of disturbances on this system shows that a cooscillating tide having a 68° phase shift is relatively insensitive with regard to possible changes in local tidal amplitudes. It may be noted in this connection that tidal amplifications of the order of two and one-half are common in many locations, as for example, Long Island Sound. The primary reason for the large range of tide in the Bay of Fundy is the unusually high range of tide at the ocean entrance. The entrance tide, 13.9 feet at Cutler, Maine, represents the tide in the Gulf of Maine which has already undergone a primary amplification due to the shoaling effect of the continental shelf (Georges Bank) at the seaward limit of the Gulf of Maine.

3. The construction of the tidal power development dikes and control structures will reduce the lateral flows from the Passamaquoddy-Cobscook Bay area. The magnitude of this disturbance can be judged by comparing the lateral tidal flows from the power development area with the longitudinal flows in the Bay of Fundy proper. In the existing, uncontrolled state, the lateral flows at their peak rate amount to 1.25 per cent of the longitudinal flow. In the developed state, the lateral inflow rate is reduced to 0.70 per cent and the lateral outflow rate to 0.45 per cent.

   It is therefore concluded that the change introduced by the power development represents a considerable reduction in the present disturbance and therefore a decrease in the loading on the system. Such a condition has been shown to favor a slight increase in the local tidal amplitude, if indeed any change can be detected in view of the slight flow modifications demonstrated for the Bay of Fundy tidal system.

4. The present study has brought out the fact that much of our analytical knowledge of tidal phenomena is lacking in experimental confirmation. It would be especially desirable to undertake a generalized study of cooscillating tidal systems and to investigate the separate effects of geometric configuration and boundary resistance, which must of necessity be lumped together. A fundamental investigation of these basic factors and their influence on the total damping in systems which may approach resonance would greatly enhance the detailed understanding of the tidal phenomena in question.

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